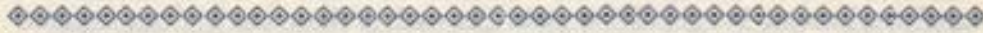


Egyptian Number Systems

Activity A Read and Discuss the following excerpts from Joseph.

3 The Beginnings of Written Mathematics: Egypt



THE URBAN REVOLUTION AND ITS AFRICAN ORIGINS

In the last chapter we began our examination of early evidence of mathematical activity with an artefact found in the middle of Africa. For the next stage of our journey we remain on the same continent, but move north to Egypt. Egypt is generally recognized as the homeland of one of the four early civilizations that grew up along the great river-valleys of Africa and Asia over five thousand years ago, the other three being in Mesopotamia, India and China. Egyptian civilization did not emerge out of the blue as a full-blown civilization without any African roots. Carbon dating of the remains of barley and einkorn wheat found at Kubbania, near Aswan in Upper Egypt, shows the beginnings of agriculture to have been around 16 000 BC, and this evidence is supported by the large concentrations of agricultural implements from around 13 000 BC found during the UNESCO-led operations to salvage the ancient monuments of Nubia.

Although there are no tangible traces of the origins of these neolithic communities, recent archaeological discoveries indicate that they may have belonged to groups from the once fertile Sahara region who were forced to migrate, initially to the areas south and east, as the desert spread. So, just as Egypt was a 'gift of the Nile' (in the words of Herodotus), the culture and people of Egypt were at least initially a 'gift' of the heartlands of Africa, the inhabitants of which were referred to then as 'Ethiopians'. This is borne out by the historian Diodorus, who wrote around 50 BC that the Egyptians 'are colonists sent out by the Ethiopians . . . And the large part of the customs of the Egyptians . . . are Ethiopian, the colonists still preserving their ancient manners' (Davidson, 1987, p. 7).

It is important that the African roots of the Egyptian civilization are emphasized so as to counter the still deeply entrenched view that the ancient Egyptians were racially, linguistically and even geographically separated from

Africa.* Recent work, well summarized by Bernal (1987) and Davidson (1987), lays bare the flimsy scholarship and ideological bias of those who persist in regarding ancient Egypt as a separate entity, plucked out of Africa and replanted in the middle of the Mediterranean Sea.

*Davidson (1987, pp. 1–2), writing about public reactions to a television series that he presented on the history of the Africans, points out that what a number of viewers in Europe and North America found particularly difficult to accept were the 'black' origins of the ancient Egyptians: 'To affirm this, of course, is to offend nearly all established historiographical orthodoxy.'

NUMBER RECORDING AMONG THE EGYPTIANS

There is an impression, fostered (no doubt inadvertently) by many textbooks on the history of mathematics, that only one scheme of numeration was used in ancient Egypt: the hieroglyphic. This impression is quite consistent with a view of Egyptian civilization as stable and unchanging, with mathematics primitive yet sufficient to serve the economic and technological needs of the time. The truth is very different from this view. It is possible to distinguish three different notational systems – hieroglyphic (pictorial), hieratic (symbolic) and demotic (popular) – the first two of which made their appearance quite early in Egyptian history. The hieratic notation was employed in both the Ahmes and Moscow Papyri. The demotic variant was a popular adaptation of the hieratic notation and became important during the Greek and Roman periods of Egyptian history.

Activity B. Exploring Egyptian Number and Operations

The following activity pages were created for an Ethnomathematics course for in-service math teachers. Look them over and do a few of the problems on activity sheet 1 and 2 to familiarize yourself with the system and what addition and subtraction look like with the hieroglyphics. Sheet 3 asks you to explore how they multiplied.

After you complete these activities, answer the following questions.

What questions do you have about the mathematical ideas? The hieroglyphics and operations with them? the Egyptians?

Egyptian Numeration

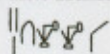
Activity 1

You are part of a team that has just discovered part of an ancient Egyptian Papyrus, and you are trying to decipher the hieroglyphics. You are confident that the information relates to numbers.

Here are eight rows of the Papyrus.



1. Try to make some conjectures about what the symbols mean and how the Egyptian Numeration system works.
2. What does this table represent?
3. Based on your ideas, write the following numbers in hieroglyphics.
 - a. 47
 - b. 83
 - c. 275
 - d. 743
4. Write 105 in hieroglyphics.
5. Is the Egyptian Numeration system dependent on place value? Explain.
6. What number is presented below? Explain your thinking.




Adapted from:

- *The Crest of the Peacock: Non-European Roots of Mathematics* by George Gheverghese Joseph, ©1991 by Princeton University Press
- Materials from Phil Straffin for Math 103: Cultural Views of Mathematics

Egyptian Numeration

Activity 2

Here is a chart of the hieroglyphic symbols used in Egyptian Numeration.

Symbol							
Name	Astonished Person	Polliwog	Pointing Finger	Lotus Flower	Scroll	Heelbone	Staff
Value	1,000,000	100,000	10,000	1000	100	10	1

- Use hieroglyphics to compute the following. Explain your method.
 - $13 + 45$
 - $25 + 34$
 - $25 + 35$
 - $47 + 105$
- Use hieroglyphics to compute the following. Explain your method. Pay close attention to "borrowing."
 - $98 - 33$
 - $476 - 123$
 - $105 - 47$
 - $562 - 486$
- Show how to double 105 using hieroglyphics.

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Egyptian Numeration

Activity 3

The Egyptian algorithm for multiplication is based on a continual doubling process. For this reason, it is sometimes referred to as the “halving/doubling” method.

Here is an example (*the crest of the peacock*, 64) of the Egyptian method to multiply 17 by 13.

$\begin{array}{l} \text{ } \\ \text{ } \end{array} \curvearrowright$	
$\begin{array}{l} \text{ } \\ \text{ } \end{array} \curvearrowright \curvearrowright$	
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$\begin{array}{l} \text{ } \\ \text{ } \end{array} \curvearrowright \curvearrowright \curvearrowright \curvearrowright$	$\begin{array}{l} \text{ } \\ \text{ } \end{array}$
$\begin{array}{l} \text{ } \\ \text{ } \end{array} \curvearrowright \curvearrowright \curvearrowright \curvearrowright \curvearrowright \curvearrowright \curvearrowright \curvearrowright$	
$\begin{array}{l} \text{ } \\ \text{ } \end{array} \curvearrowright \curvearrowright \curvearrowright \curvearrowright$	$\begin{array}{l} \square \\ \text{ } \end{array} \curvearrowright$

1. Analyze the example by completing the following:
 - a. Show where the numbers 13 and 17 appear in the hieroglyphics.
 - b. Explain the first column of the calculation.
 - c. Explain the second column of the calculation.
 - d. Which elements of the second column must be added to equal 13?
 - e. Add the corresponding elements of the first column.
 - f. Check your result by using “conventional” multiplication.

2. Try some other problems using this method (you do NOT have to write in hieroglyphics).

a. 27×11	b. 13×41	c. 12×13
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3. Try one (or more) of the problems above, but use the other number as the multiplicand. Does this method still work?

4. Will this process for multiplication always work? Explain.

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- *The Crest of the Peacock: Non-European Roots of Mathematics* by George Gheverghese Joseph, ©1991 by Princeton University Press
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EXTENSION QUESTIONS

5. Look at the 7th and 8th rows of the tablet, in figure 2. We can use the notation below to describe these rows.

Column 1	Column 2
7	1, 3
8	1, 12

Using this notation, find the value in column 2 when column 1 is 20.

6. Using the notation above, find the value of the three digit number 1, 4, 21. Justify your answer.

Egyptian Numerals

Homework

1. Show how an Egyptian scribe might have subtracted 871 from 1023. Pay careful attention to how the “borrowing” would work. Do your calculation in Egyptian hieroglyphics.
2. Show how an Egyptian scribe would have used the “halving/doubling” method to multiply 312×27 .
3. Below is the solution to problem #79 from the Ahmes (Rhind) Papyrus. The translations of the hieroglyphs are at the right.
 - a. Decode the numbers.
 - b. Ahmes made one misprint. What is it? What are these numbers?

(houses)

(cats)

(mice)

(sheaves of wheat)